



A Modified Goldak Heat Source for Enhanced Wire Arc Additive Manufacturing Modeling

Validated by experimental local temperature and cross-sectional macrographs

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Abstracts

This paper presents a modified Goldak heat source (GHS) model applied to duplex stainless steel (DSS) wall-of-weld (WoW) and AA5356 aluminum in wire arc additive manufacturing (WAAM) using gas metal arc welding. Although prior modifications to the GHS exist, none have addressed the actual arc origin following arc initiation. Consequently, maintaining realistic parameter ranges in the standard GHS often results in artificially high temperatures, inconsistent interlayer boundaries, or large relative errors compared to experimental data. In this study, an arc origin offset (as a new Goldak parameter, Z_o) is introduced in the double ellipsoidal heat source equations, reflecting the actual location of peak heat input. As a result, Z_o improves the prediction of solid-liquid phase transitions of the melt pool cross-sectional geometry validated by the macrographs. A thermal study optimizing this parameter, alongside a parametric study of this Modified GHS (MGHS), is first implemented on DSS-WoW and subsequently on the AA5356-WoW. The standard GHS and the MGHS are compared in each case and validated against experimental results and DSS macrographs. The results demonstrate how Z_o enhances standard GHS and, more importantly, adjusts it to WAAM beyond a single weld bead. Additionally, an experiment conducted by the authors on an AA1100 Tube-to-Weld using a moving pyrometer (MIKRON-ME-PI 140) accurately confirmed the MGHS model. To estimate the Z_o value in a more generalized way, a novel dimensionless number was developed to empirically relate Z_o to five alloys and process parameters. This enables researchers and engineers to select the desired Z_o value.

Keywords

- Modified Goldak Heat Source (MGHS)
- Solid-Liquid Phase Change
- Interlayer Boundaries
- Arc Origin Offset
- Double Ellipsoidal Heat Source
- Macrograph
- Wire Arc Additive Manufacturing (WAAM)

Introduction

Applications of directed-energy-deposition (DED) have been recently employed to fabricate high-entropy alloys, such as AlCoCrFeN studied by Oyerinde et al. (Ref. 2) for microstructural evolutions or high-strength alloys like Ti-6Al-4V alloy studied by K. Wijesinghe et al. (Ref. 3) for the investigation of microscopic deformation under in-situ uniaxial tensile testing. Wire arc additive manufacturing (WAAM) is a cost-effective method of DED. In WAAM processes, different heat sources significantly impact the quality and properties of the deposited parts. Elmer et al. (Ref. 4) compared laser, arc, and electron beam additive manufacturing (AM), highlighting WAAM's intermediate energy input and suitability for layer-by-layer fabrication with tailored properties. Applications of WAAM have extended to high-performance alloys due to its multi-functionality. Ding et al. (Ref. 5) reviewed the advancements and challenges in WAAM, emphasizing the need for better thermal management and stress mitigation. Hejripour et al. (Ref. 6) conducted a thermal study to characterize WAAM-duplex stainless steel (DSS). Hejripour and Aidun (Ref. 7) investigated the effects of processing parameters on the geometry, microstructure, and mechanical properties of WAAM-Inconel 718, identifying challenges such as grain growth orientation and intermetallic formation. Although improving the mechanical properties of the alloy was not the focus of their work, the way it was studied by R. Mazruee Sebdani et al. (Ref. 8) on SS 17-4 PH through a combination of Ultrasonic-Impact-Treatment and powder-bed-fusion

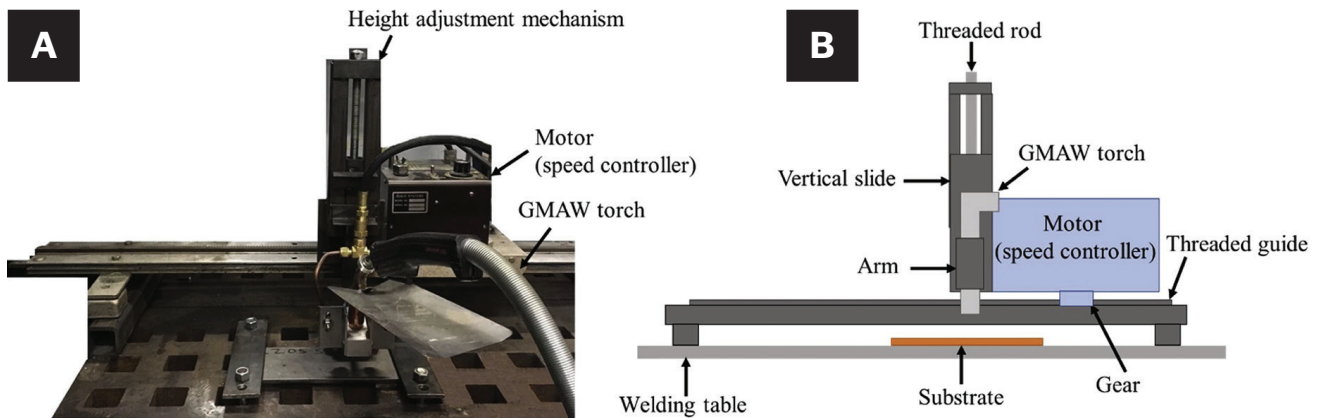


Fig. 1 — A — Experimental setup for building the WoW; B — the schematic illustration of the setup, courtesy of Hejripour et al. (Ref. 6).

Table 1 — The Chemical Composition of the Filler Wire and Substrate (wt-%)

Metal	Ni	Cr	Nb	Mo	Ta	Al	C	Mn	Si	P	S	N	Cu	Fe
Substrate 2205	6.5	23.0	—	3.25	—	—	0.03	2.00	1.00	0.03	0.02	0.14	—	Balance
Wire ER2209	8.5	22.9	0.01	3.1	<0.001	0.01	0.01	1.7	0.51	0.02	<0.01	0.16	0.09	Balance

process, their study demonstrated that WAAM parts exhibit consistent mechanical properties despite these challenges. The gas metal arc welding (GMAW) heat source is the most straightforward to integrate into WAAM applications. Norrish (Ref. 9) extensively studied the evolution and advancements in GMAW and reviewed the metal-transfer mechanisms and critical control variables that influence process performance. Ibrahim et al. (Ref. 10) investigated the influence of GMAW parameters on weld penetration, microstructure, and hardness in a 6-mm-thick mild steel using robotic-GMAW. Controlling the microstructure in WAAM is challenging due to the complex thermal analyses, which directly affect phase formation. Hence, the more accurate the numerical model of the temperature distribution, the more capabilities it provides for exploitation in multiple WAAM-related numerical analyses, such as predicting the austenite-to-ferrite transformation (Ref. 11), residual stress analysis (Refs. 12, 13), and local fluid flow study of the weld pool (WP) (Ref. 14).

Most metal AM processes utilize a heat source, such as a laser, electron beam, or arc, during the fabrication process to melt or fuse the material. Nascimento et al. (Ref. 15) conducted a comprehensive cataloging of heat source models and their advancements over the past two decades. Furthermore, their research provided a statistical analysis of modeling methodologies, publication trends, and a chronological timeline of modeling developments. Based on this analysis, the Gaussian heat source distribution emerged as the most widely used method, comprising 35.2% of the reviewed studies, followed by the double ellipsoidal model at 21.8%. They also mentioned that Goldak et al. (Ref. 1) were

pioneers in introducing a three-dimensional heat source model. In another review by Yang (Ref. 16), Goldak's double-ellipsoidal model was highlighted as one of the most widely utilized for fusion welding thermal models. Although this could be less suitable for processes like keyhole-laser or electron-beam-welding, or arc welding in deep, narrow grooves (Ref. 17), the approach remains highly effective for WAAM using GMAW, provided the parameters are carefully calibrated in numerical simulations to match the relevant experimental geometries. However, achieving such accurate modeling remains a challenge and requires careful computations to ensure reliable thermal field predictions. To address this challenge, there are relatively few works that focus on optimizing and tuning parameters based on experiments, previous publications, and numerical methods. In this context, a thorough study, although applied to a laser heat source rather than arc welding, was conducted by Xiaofeng Hu et al. (Ref. 18) to review several laser heat source models. They developed a parametric optimization method to calibrate Goldak parameters. Their approach employed genetic algorithms (GA) to minimize discrepancies between experimental and simulated melt pool boundaries, achieving less than 2% error in theoretical cases. However, they have not provided any data on the computational time for their proposed method, nor have they compared the optimal parameters for the initial layer(s) where the substrate is not preheated with those for the upper layers that have been heated prior. Additionally, their study only considered the heat transfer equation, including the latent heat term. Other

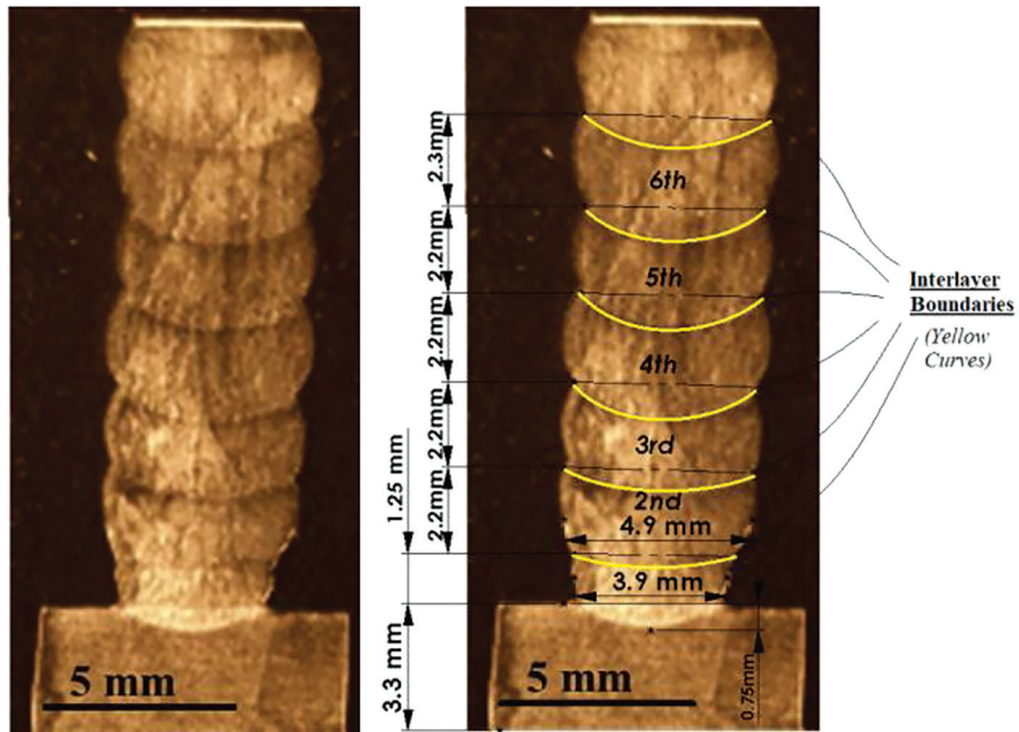


Fig. 2 — Macrograph of the (first six layers) sectioned WAAM-DSS (Ref. 6). The measure of 5mm marked on the reference figure was used to scale all other dimensions.

physics, such as solid mechanics, were neither coupled nor accounted for in a decoupled approach.

The challenge of adjusting the heat source parameters to achieve an accurate thermal model in an arc-based heat source was addressed by Changmin Pyo et al (Ref. 19). GA and its combined versions, like the particle swarm optimization in association with GA in a hybrid model, have been widely used in science and engineering (Ref. 20); Changmin Pyo et al. used a multi-island GA to optimize the parameters by comparing the HAZ boundary from experimental Bead on Plate (BOP) welding with simulations. However, their study was limited to a single-bead GMAW process. Furthermore, the authors did not provide details of the governing equations. There was also no explanation of either the error or the interaction between the GA optimization process and the FEM simulations. The distance-to-heat-source variable was neither explicitly defined in any equation nor related to the control variables or objective function in GA. Additionally, the HAZ was defined as a target using mild steel references from Layus et al. (Ref. 21), which could be inconsistent with the SS400 substrate welded with AWS A5.29, as these materials exhibit different microstructures in the HAZ region.

The thermal models are not limited to only FEMs. Bjelić et al. (Ref. 22) developed a numerical model using the finite-difference method (FDM) to simulate heat transfer during the welding of thin sheets. Although GMAW-adjusted heat source parameters on a thin sheet (2D-mode) cannot be applied to thick substrates within a 3D nature, they applied a simulated annealing optimization method to calibrate uncertain heat source parameters, including arc efficiency, emissivity, and enhanced conductivity, during the welding of P355GH steel. Chergui et al. (Ref. 23) developed a finite element approach

for WAAM that incorporates a standard Goldak heat source model adapted for gradual material deposition along the toolpath. In this paper, similar to all works published in the literature on non-fluid flow-related thermal analyses, the focus is on evaporation (Ref. 24) is considered negligible; moreover, since the geometry is already simplified based on the experiment and the focus of the studies is not the local region of WP, local effects, including fluid flow, free surface (Ref. 25), gravity effects, Marangoni flow (surface tension gradient, [Ref. 26]), also buoyancy forces and electromagnetic fields (Ref. 26) are disregarded.

Based on an extensive review of the literature, the widely adopted Goldak heat source model and its associated challenges reveal a critical gap: no studies have incorporated a realistic arc origin. Furthermore, existing thermal models for WAAM lack validation through comparisons of cross-sectional phase change predictions with macrostructural layer analyses. This analysis also involves the shape of the interlayer boundary. To address these gaps, a novel correction term, denoted as the arc origin offset (Z_o), was introduced into the standard Goldak heat source equations along the wire feed direction (z-axis). This offset enhances the physical accuracy of the Goldak heat source model, allowing for the adjustment of its parameters to realistic ranges and yielding more precise temperature distributions, as well as an improved representation of interlayer boundaries in WAAM. The application of the modified Goldak heat source (MGHS) to DSS and AA5356 validated these improvements. Additionally, an AA1100 tube-to-weld (ToW) experiment, designed by the authors to test the MGHS model on tubular geometries and simulate a moving pyrometer using a moving point, successfully validated the model. Together with additional MGHS

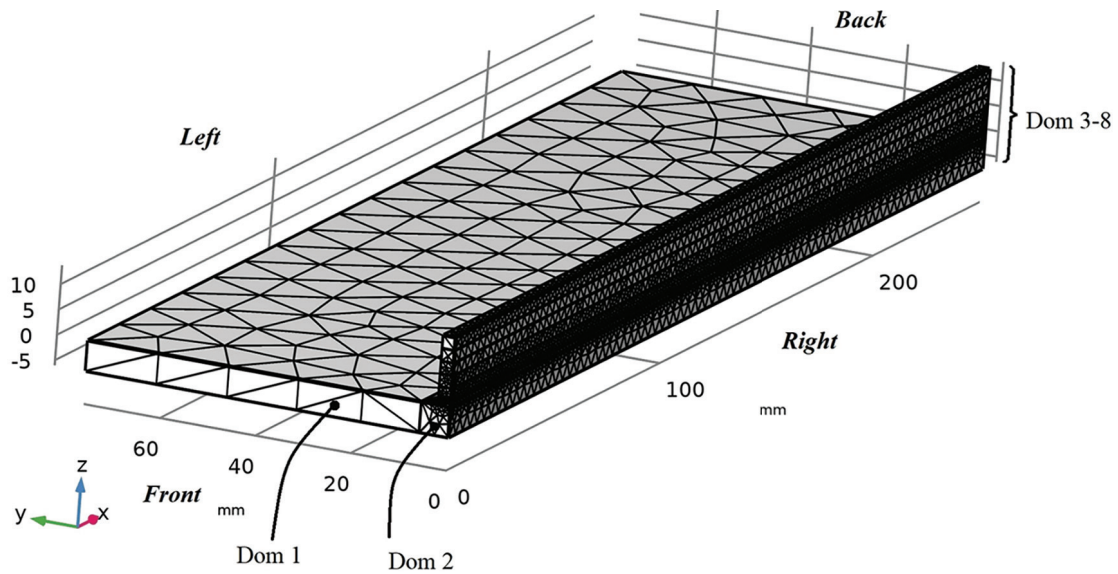


Fig. 3 — Domains (with symmetry BC), medium mesh (mesh type 1) for the parametric study and optimization process.

Table 2 — Boundary Conditions (BC)

BC	Equation	Description
Thermal Insulation and Symmetry	$\mathbf{n} \cdot \mathbf{k} \nabla T = 0$	Applied to front/back faces (Fig. 3), assumed insulated due to clamping.
Surface to Ambient Radiation	$\mathbf{n} \cdot \mathbf{k} \nabla T = \varepsilon \sigma (T_{amb}^4 - T^4)$	Applied to surrounding surfaces, $T_{amb} = 298.15K$, $\varepsilon = 0.9$ (Dom 2-8), $\varepsilon = 0.7$ (Dom 1), (Ref. 28).
Convective Heat Transfer	$\mathbf{n} \cdot \mathbf{k} \nabla T = h(T_{amb} - T)$	Two different domain dependent temperature dependent expressions are considered for convective heat transfer coefficient h .

models' results validated by experimental data from other alloys (ER90S-B3 and 8Cu-0.4Mn-0.22Cd-0.16Ti [ZL205A]), this experiment enhanced the robustness of a user-friendly curve fit for the Z_o parameter using a novel dimensionless number developed by the authors that relates Z_o to five different alloys and process parameters, enabling researchers to select the desired Z_o value.

Experimental Procedure for DSS

The experimental data for DSS were obtained from prior experiments conducted by our research group under Dr. Daryush Aidun's supervision, as reported in Ref. 6, which provided thermocouple measurements for model validation. In this study, the first experimental setup (Fig. 1) and data resources were extracted from the thermal study done by F. Hejripour et al. (Ref. 6). Although more advanced thermal monitoring instruments, such as the FLIR One Pro camera,

have been employed at times in welding experiments (Ref. 27), the type-K thermocouple was the temperature measurement tool in their experiment. The chemical composition of the filler wire and substrate is shown in Table 1.

In the interest of computational time and to ensure better consistency with their experimental temperature curve, only the first six layers of deposition were taken into account. Also, the symmetry condition was assumed at the middle cross-section across the length, so the substrate was modeled as a 250 mm × 76 mm × 6 mm plate. The initial WAAM process parameters, voltage, current, travel speed (TS), and wire feed speed (WFS) for the model were set to 25V, 155A, 12 mm/s, and 147 mm/s, respectively, matching the reference experiment.

Some of the Goldak parameters were adjusted with the macrograph of the cross-sectional geometries of the layers and substrate from the reference experiment (Fig. 2).

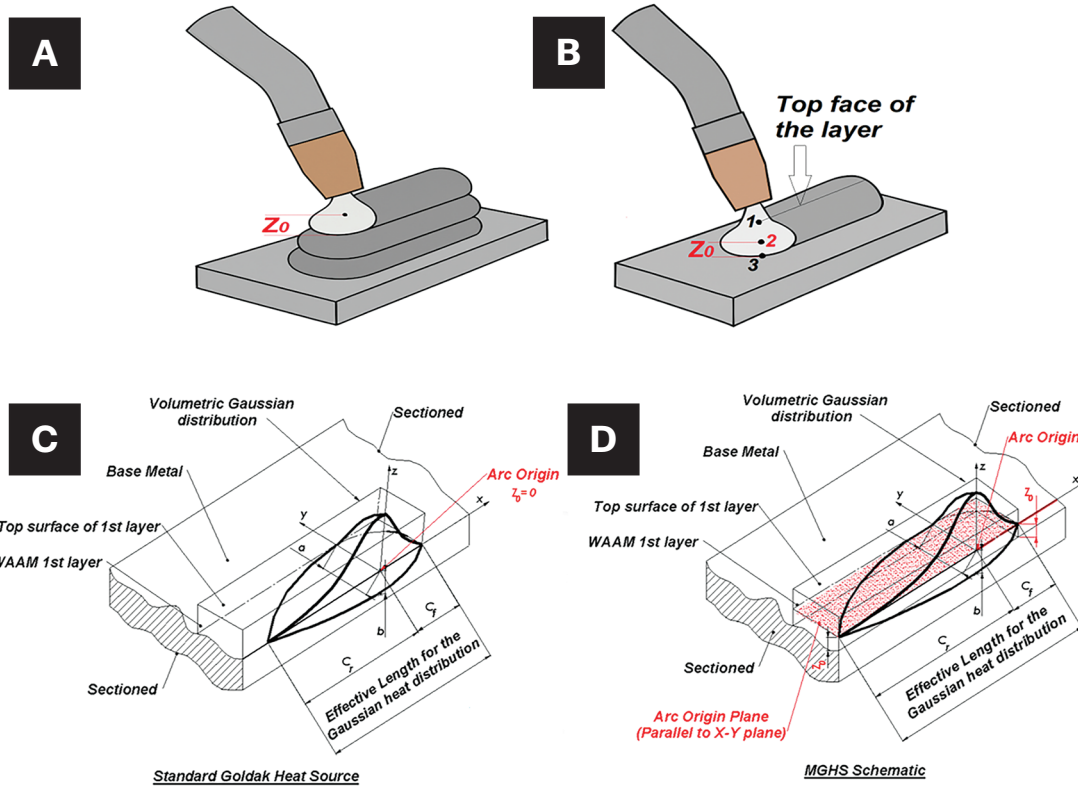


Fig. 4 – Double ellipsoidal power density distribution in MGHS with Z_0 parameter.

Table 3 – Heat Source Parameters

Parameter	Description
Heat Fractions (f_f and f_r)	Portions of total heat in front and rear quadrants; $f_f + f_r = 2$.
Semiaxis Lengths (a , b , c_f , c_r)	a : y-direction, b : z-direction, c_f : front portion (x-axis), c_r : rear portion; adjusted per quadrant; a and b : set to values around bead's width and penetration depth for realistic physics (all in mm).
Q_{arc}	Energy Input (W)
Z_0	Arc origin offset (mm), (Novel parameter)

Numerical Model on DSS

Heat Transfer

A common heat transfer equation in solids is the following equation:

$$\rho C_p \left(\frac{\partial T}{\partial t} + \mathbf{u}_{trans} \cdot \nabla T \right) + \nabla \cdot (\mathbf{q}_{cond}) = q_{arc} \quad (1)$$

ρ is the density (kg/m^3),

C_p is the specific heat capacity ($\text{J/kg} \cdot \text{K}$),

T is the absolute temperature (K),

$\mathbf{u}_{trans}$ (m/s) is the velocity vector of translational motion, which equals zero in this study since no bulk motion exists in the substrate and additive layers, nor is the local fluid flow considered,

q_{cond} is the heat flux by conduction (W/m^2), which is $-k \nabla T$, and k is thermal conductivity ($\text{W/(m} \cdot \text{K)}$)

q_{arc} is the volumetric heat source (W/m^3) defined as $q_r(x, y, z, t)$ and $q_f(x, y, z, t)$ in Equations 2 and 3.

Table 4 — Lower Bounds (LBs) and Upper Bounds (UPs) for the Control Variables (DSS)

Control Variable	1 st layer's LB	1 st layer's UP	Upper layers' LB	Upper layers' UP
	6 mm (0.236 in.)	10 mm (0.394 in.)	9 mm (0.351 in.)	15 mm (0.591 in.)
	0 mm (in.)	1 mm (0.0394 in.)	0 mm (in.)	2 mm (0.0787 in.)

The coupling of the heat transfer equation with thermal expansion and stresses is included only in the final fine-mesh simulations to confirm the DSS model, enabling future residual stress analysis and solid-state phase transformation simulations, which are beyond the scope of this paper. Figure 3 shows the model's domains with a medium mesh, which include a coarse tetrahedral mesh for the large domain of the substrate (Dom 1) away from the WAAM region and an extra-fine tetrahedral mesh for other domains, consisting of 35,957 domain elements, 14,227 boundary elements, and 2580 edge elements.

The boundary conditions are as shown in Table 2. σ is the Stefan-Boltzmann constant: $\sigma = 5.670374419 \times 10^{-8}$ (W/m²K⁴). So, the boundary condition $\mathbf{n} \cdot \mathbf{k} \nabla T = h(T_{amb} - T) + \varepsilon \sigma (T_{amb}^4 - T^4)$ will be applied to all surrounding faces of MGHS models, except where symmetry or insulation is assumed or at the local weld pool, which is governed by the deposition function and element activation. Convective and radiative terms are updated each time step.

Heat Source

A common volumetric Gaussian-distributed heat source, used in welding simulations, is the standard Goldak heat source (Ref. 1) that defines how the heat is applied to the workpiece. The power density distribution along the longitudinal axis for the MGHS is depicted in Fig. 4C. The power density distribution for the front quadrant of the heat source is given by Equation 2:

$$q_f(x, y, z, t) = \frac{6\sqrt{3} f_r Q_{arc}}{abc\sqrt{\pi^3}} \exp \left(-3 \left(\frac{[x + v(\tau - t)]^2}{c^2} + \frac{y^2}{a^2} + \frac{(z - z_0)^2}{b^2} \right) \right) \quad (2)$$

and Equation 3 for the rear quadrant:

$$q_r(x, y, z, t) = \frac{6\sqrt{3} f_f Q_{arc}}{abc\sqrt{\pi^3}} \exp \left(-3 \left(\frac{[x + v(\tau - t)]^2}{c^2} + \frac{y^2}{a^2} + \frac{(z - z_0)^2}{b^2} \right) \right) \quad (3)$$

$Q_{arc} = \eta VI$, V is the voltage and I is the current. v is the travel speed (TS). The heat source parameters are as shown in Table 3. τ is the lag time defined in Ref. 1. The main term commonly used in the literature is $x - vt$, since τ is not a physical variable and is not explicitly solved in the equation,

Table 5 — Initial Values (DSS)

Parameter / BC	Value
a	1.85 mm (0.0728 in.) (1 st layer), 2.45 mm (0.096 in.) (upper layers)
b	0.75 mm (0.0295 in.) (1 st layer), 2.1 mm (0.0827 in.) (upper layers)
c_f	~ half bead width (per Goldak et al., Ref. 1)
c_r	Table 4
Z_0	0
f_f, f_r	$f_f = 1.4, f_r = 0.6$, alternatives: $f_r = 1.52, f_f = 0.48$ (Ref. 33), $f_r = 1.5, f_f = 0.5$ (Refs. 31, 32)
h_{weld}	$24.1 \times 10^{-4} \times \mathcal{E} T^{1.6}$, Rykalin (Ref. 33)
$h_{nonwelded}$	$T/20$, inspired by Rykalin (Ref. 33)

but is instead chosen to define the initial position of the heat source in the moving coordinate system relative to the fixed coordinate system, which is implemented in either coding or commercial software as an input in UDF (user-defined function). It becomes particularly important in simulation, where the heat source moves back and forth, as in WAAM.

In a standard Goldak heat source without Z_0 , locations 1 (on top of the layer) or 3 (on the substrate) are considered as the arc origin (Fig. 4B); however, location 2 is the realistic origin of the arc proposed in this paper. It defines an arc origin offset (Z_0) in Equations 2 and 3, as highlighted in red. Physically, the peak thermal energy deposition does not occur exactly at the geometric interface where the arc visibly contacts the surface. In GMAW-based WAAM, the wire is continuously fed into the weld pool at a specific angle and velocity. This causes the actual interaction zone between the arc and the substrate to shift upward. This spatial offset along the z -axis (the build direction) means that the standard Goldak model, which assumes the heat source origin centered at the surface (Fig. 4C), fails to represent the true volumetric energy distribution. Introducing the Z_0 parameter (Fig. 4D) into the heat source formulation allows for this vertical displacement of the arc's effective origin, aligning the mathematical model with the physical reality of droplet-based energy and mass transfer.

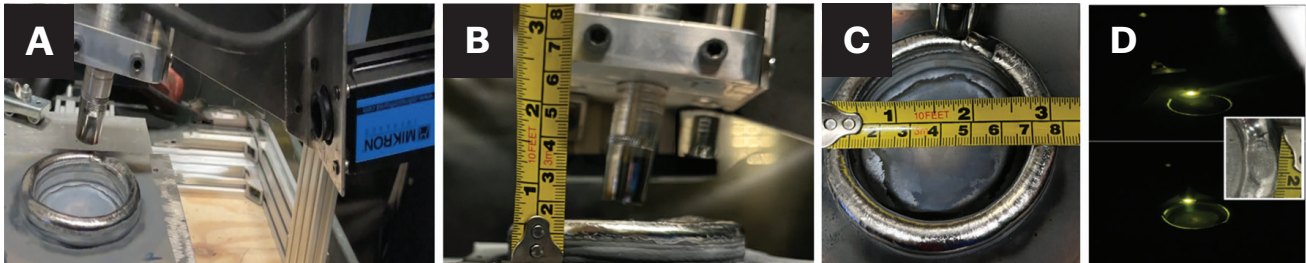


Fig. 5 – WAAM AA1100 cylindrical tube of wall (ToW), conducted by the authors.

Table 6 – DSS WoW's Process Parameters (Experiment by Ref. 6)

Welding Current (A)	Welding Voltage (V)	Travel Speed	Wire Feed Speed	Idle-Time (s)
155	25	12 (mm/s) (28.4 in./min)	147 (mm/s) (347.3 in./min)	0

Table 7 – AA5356 Aluminum WoW's Process Parameters (Experiment by Ref. 23)

Welding Current (A)	Welding Voltage (V)	Travel Speed	Wire Feed Speed	Idle-Time (s)
80	12	10 (mm/s) (23.622 in./min)	5 (m/min) 83.33 (mm/s) (196.85) in./min)	2

Table 8 – AA1100 Aluminum ToW's Process Parameters (Experiment by Authors)

Welding Current (A)	Welding Voltage (V)	Travel Speed	Wire Feed Speed	Idle-Time (s)
160	25	2 rpm 7.54 (mm/s) (17.8 in./min)	11.43 (m/min) 90.5 (mm/s) (450 in./min)	445

The results at the end confirm the values of Z_o varying across the layers (Figs. 4A and B). In this study, since parametric analyses showed that Z_o values for layers above the second layer had minimal variation, only two Z_o values were considered for the WAAM process: one for the first layer and one for the upper layers (Table 4). This is because the substrate for the first layer is the bulky base metal. In contrast, for the upper layers, the substrate is the previously deposited layer, as each layer above the second layer has approximately the same geometry. Five different validations on five alloys (DSS2209, 5356 Al, 1100 Al, ER90S-B3, and ZL205A) incorporating the two-value Z_o assumption for layers on both WoW and ToW confirm this assumption, with all errors lower than 4%.

Parametric Study and Optimization of DSS Model

Parameters of interest including:

$\eta \cdot I$: Multiplication of arc-efficiency and electrical current

a, b, c_p, c_f, f_p, f_r : Standard Goldak parameters

Z_o : Correction term (New Parameter in MGHS)

h_{weld} and $h_{non-welded}$: Temperature-based convective heat transfer

Coefficients are tuned by parametric study starting from the term $\eta \cdot I$ and heat transfer convective coefficients, followed by the parameter optimization using COMSOL SNOPT solver (Ref. 29). The objective function (O. F.) is the least squared error (Eq. 4) representing the difference between experimental data and simulated results of temperature solved in

Table 9 — ER90S-B3 WoW's Process Parameters (Experiment by Ref. 36)

Welding Current (A)	Welding Voltage (V)	Travel Speed	Wire Feed Speed	Idle-Time (s)
255	19	10 (mm/s) 23.62 (in./min)	133.33 (mm/s) (315 in./min)	180

Table 10 — ZL205A WoW's Process Parameters (Experiment by Ref. 37)

Welding Current (A)	Welding Voltage (V)	Travel Speed	Wire Feed Speed	Idle-Time (s)
107	13.2	8 (mm/s) 18.9 in./min)	100 (mm/s) (236.2 in./min)	60

Table 11 — DSS-WoW Optimized Values of Z_o and C_r

Control Variable	1 st Layer	Upper Layers
C_r	9 mm (0.354 in.)	12 mm (0.472 in.)
Z_o	0.85 mm (0.033 in.)	1.5 mm (0.0591 in.)

each iteration, where a couple values are assigned to the control variables repetitively.

$$O.F. = \sum_{i=1}^n (T_{Sim}(c_r, z_0, t) - T_{exp}(c_r, z_0, t))^2 \quad (4)$$

where, T_{Sim} is the simulated temperature, T_{exp} is the experimental temperature, and t is time. As a future work, a WAAM thermal study incorporating a magnetic field could leverage the hybrid Moving Least Squares–Particle Swarm Optimization approach proposed by Mahmoodabadi et al. (Ref. 30) for solving a nonlinear transient convective–radiative heat transfer process. All the simulations in this paper are time-dependent (transient).

The lower and upper bounds for the control variables c_r and Z_o are given in Table 4. Initial values were set to the first layer's lower bounds.

Initial Values for the Numerical Analysis of DSS

The initial values to start the numerical modeling are given in Table 5.

While thermal expansion was not included at this stage to save time, solid mechanics to activate the elements and phase change were coupled. The material properties were chosen to be temperature-dependent, like in Ref. 6.

Experimental Methodology for AA1100 Tube of Wall (ToW)

This experiment was conducted by the authors using a robotic WAAM system to fabricate the ToW of AA1100 on a 6061 aluminum substrate, equipped with a moving pyrometer (MIKRON ME-PI 140) attached to the moving torch that continually monitors the temperature at an angular offset of $\pi/5$ behind the weld pool. This approach is more reliable than a single or a limited number of fixed thermocouples. This experiment was performed to verify the MGHS model's capability on different geometries, such as a circular tube (Figs. 5A–C), and to confirm the model through a moving point simulating the moving pyrometer. The initial values of the standard GHS parameters were tuned based on the geometry of the weld pool, as shown in Fig. 5D, and then adjusted to MGHS. The standard forehand angle is 10 deg to 15 deg from perpendicular, was taken into account as well, as seen in Figs. 5A and B.

Process Parameters for Experimental Validations of Different Alloys

As mentioned before, the validation of the MGHS equations was not limited to the WAAM-DSS experiment. Four other alloys with almost distinct material properties and process parameters were used to validate the MGHS model.

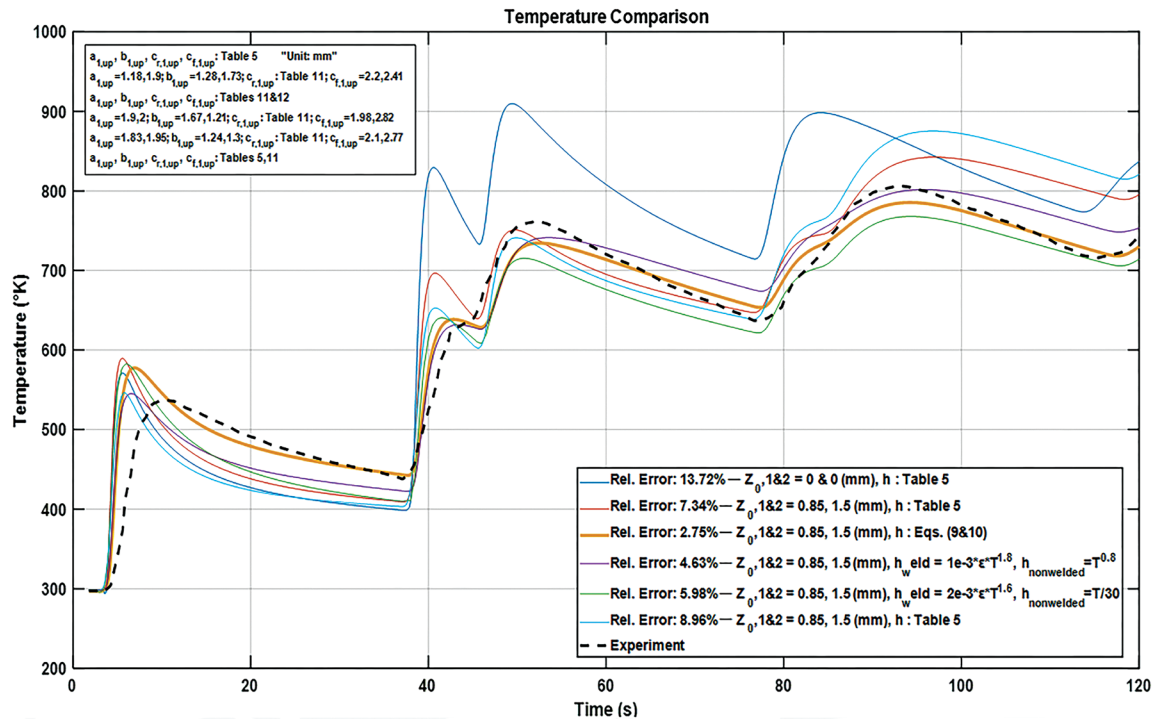


Fig. 6 — Some of the MGHS time-dependent temperature profiles at the substrate bottom location (45 mm, 0, -6 mm).

Table 12 — DSS-WoW MGHS Parameters

Parameter	1 st Layer	Upper Layers
a	1.7 mm (0.067 in.)	2 mm (0.079 in.)
b	1.2 mm (0.047 in.)	1.5 mm (0.059 in.)
c_r	9 mm (0.354 in.)	12 mm (0.472 in.)
c_f	1.9 mm (0.075 in.)	2.9 mm (0.114 in.)
f_r	1.25	1.25
f_f	0.75	0.75

The process parameters of the primary DSS model, explained under Table 1, are also tabulated in Table 6 along with other alloys.

AA5356 aluminum was fabricated by Chergui et al. (Ref. 23), and the MGHS temperature error dropped to 3.4% for the thermocouple P2 thermocouple in a zigzag strategy (ref. 23). The process parameters of the AA5356 are given in Table 7.

Apart from walls of weld, a tube of weld (ToW) of AA1100 aluminum was fabricated (Fig. 5), and the MGHS model of this ToW resulted in a 3% error. The process parameters of the AA1100 aluminum are given in Table 10 and the corresponding heat source parameters are presented in Table 8.

Table 9 presents the process parameters for the ER90S-B3 alloy. An MGHS model validated by the experiment conducted by F. W. C. Farias et al. (Ref. 36) demonstrated high accuracy, with an average error of 3.2%.

Table 10 presents the process parameters for the ZL205A alloy. Similarly, an MGHS model validated by the experiment of Z. Wang et al. (Ref. 37) showed good agreement, with an average error of 3.9%.

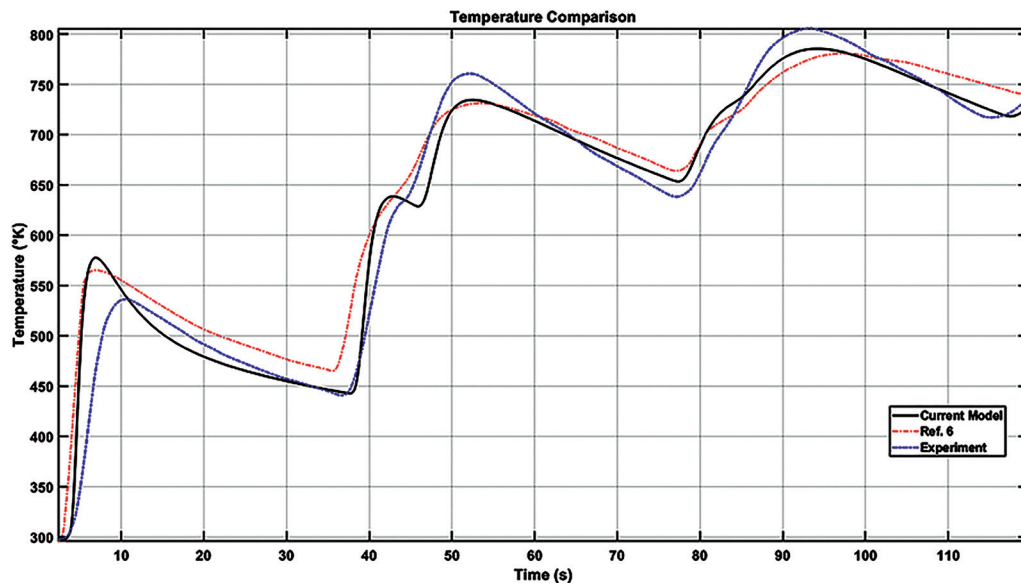


Fig. 7 – The comparison of the temperature profile by MGHS, Ref. 6 model, and experimental temperature at (45 mm, 0, -6 mm).

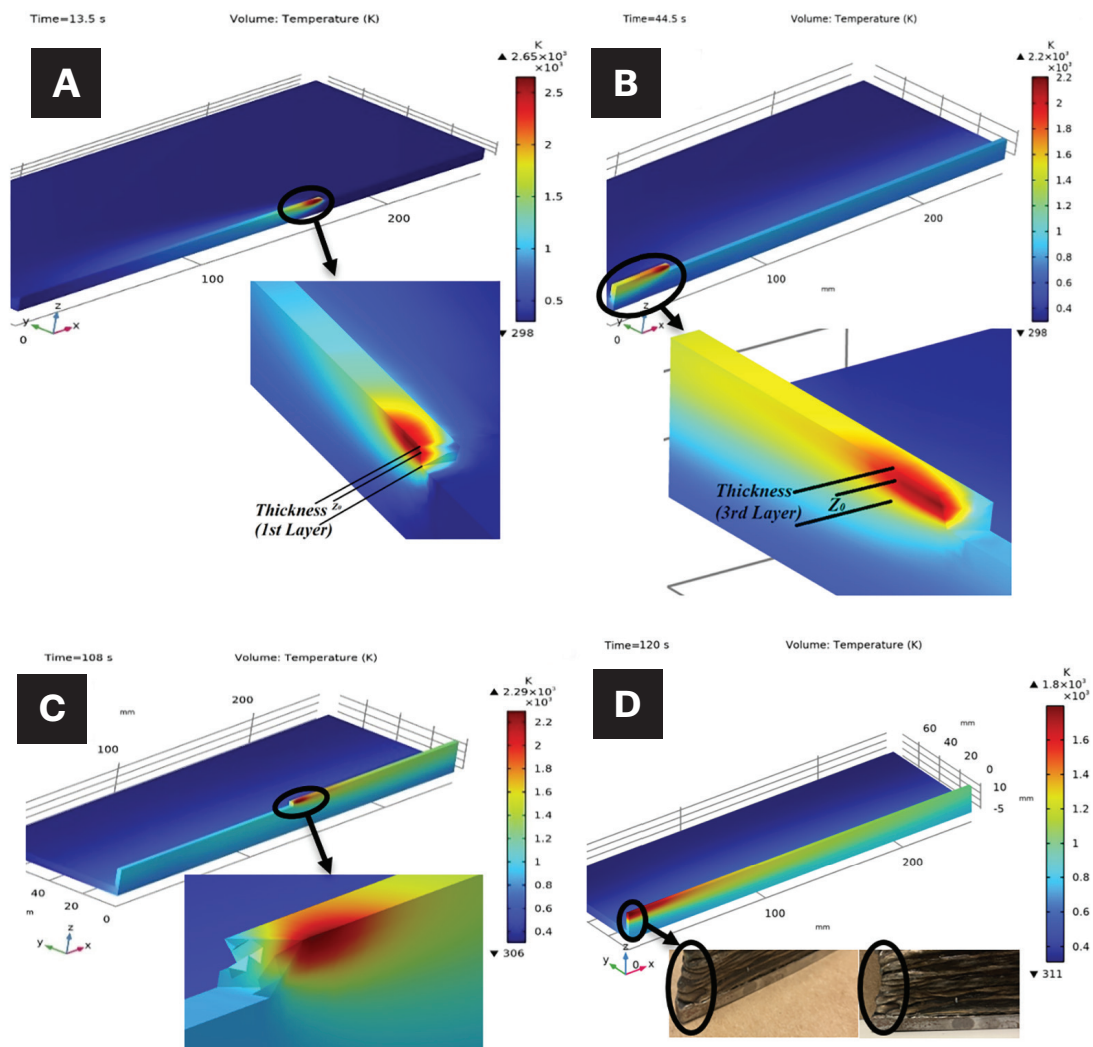


Fig. 8 – Temperature 3D plots at 13.5 s, 44.5 s, 108 s, and 120 s.

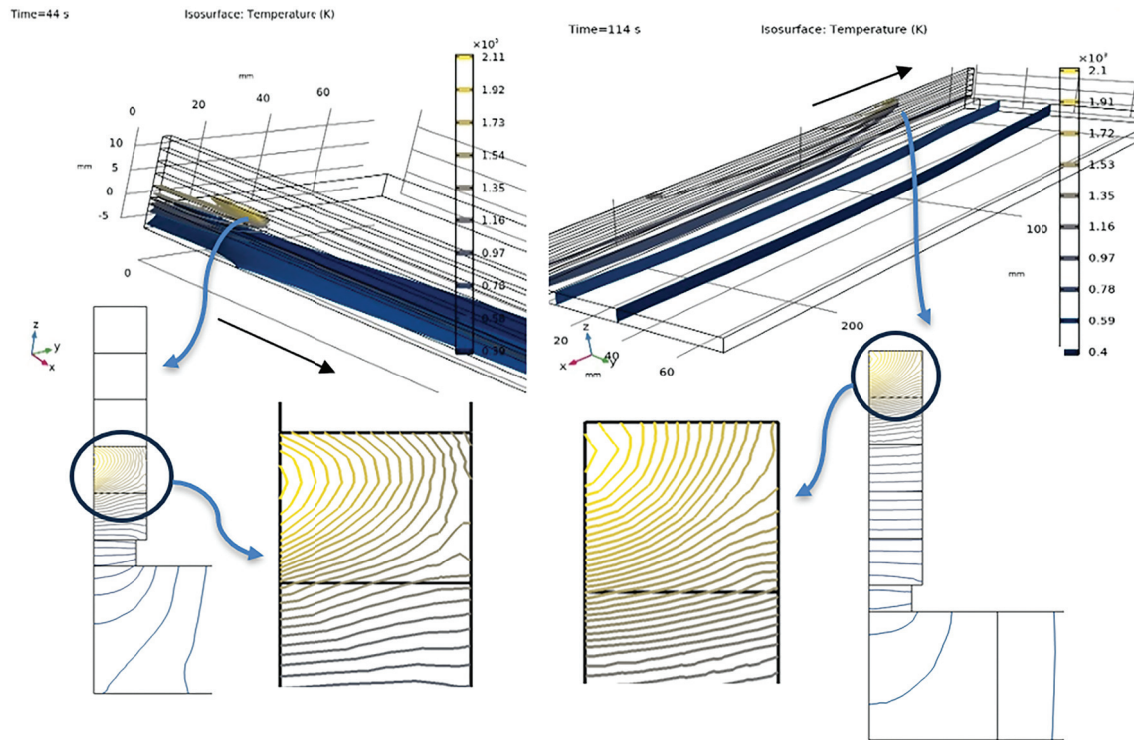


Fig. 9 – Isothermal 2D and 3D contours at 44 s and 114 s.

Results

This section presents the numerical results obtained through the parametric study and optimization of the MGHS model for the DSS. For enhanced reproducibility, the results for heat source parameters and convective heat transfer coefficients for each filler wire alloy are consolidated in Appendix B, including their variations across travel speed and wire feed speed.

Table 11 shows the optimized values of the two parameters of interest (Z_o and c_p) whereas table 12 lists the final values tuned through many simulations by the MGHS for the heat source parameters on the DSS.

Some of the temperature profiles resulting from MGHS on the WAAM-DSS (Fig. 6) derived from multiple simulations showed different errors. The one with the highest error in the figure (13.72%) is the Standard Goldak model. Figure 7 shows a comparison of the temperature curve of the MGHS with that of the model using a surface Gaussian heat distribution in the DSS experiment (Ref. 6) and the temperature profile measured by the thermocouple. Each peak in Figs. 6 and 7 also counts as one thermal cycle.

Figure 8 depicts the 3D contour plots of the temperature distribution across different layers while the elements are being deposited at different time steps, as the heat source moves back and forth. The weld pool and thicknesses of the first and second layers and corresponding Z_o in the weld pool are also highlighted in Figs. 8A and B. Figures 8C and D show the weld pool at the middle of the sixth layer and the end of it, respectively. Figure 9 illustrates the 3D isothermal contour plots across different layers and time steps as the heat

source moves back and forth. The arrows at each time step show the travel direction. At $t = 99$ s, the end of the fifth layer and start of the sixth can be observed as the arrow implies. Some interesting side views of the moving heat source are also illustrated in Fig. 10.

The interlayer boundary shape analysis, resulting from the comparison of the phase change validation and the cross-sectional macrograph, is presented in Figs. 11 and 12. The right-hand side of the graph comparison in Fig. 11 corresponds to the right half section of the sample from Fig. 2, and the left-hand side is the left half section of the simulation, both from the substrate and the first layer. Note that the macrograph has been scaled on the COMSOL phase change plots. Figure 12 illustrates a similar validation to that observed in Fig. 11 for the upper layers.

Temperature Profile and Validation of A5356 WoW

Figure 13 shows the temperature profiles for AA5356 aluminum WAAM, comparing experimental data by Chergui et al. (Ref. 23) with their model and the proposed MGHS and standard GHS by the authors of this paper.

Temperature Profile and Validation of 1100 Al Tube

The blue curve in Fig. 14 represents the temperature profile obtained from the MGHS model for the deposition of an AA1100 tube on a 6061 substrate. Only two layers were con-

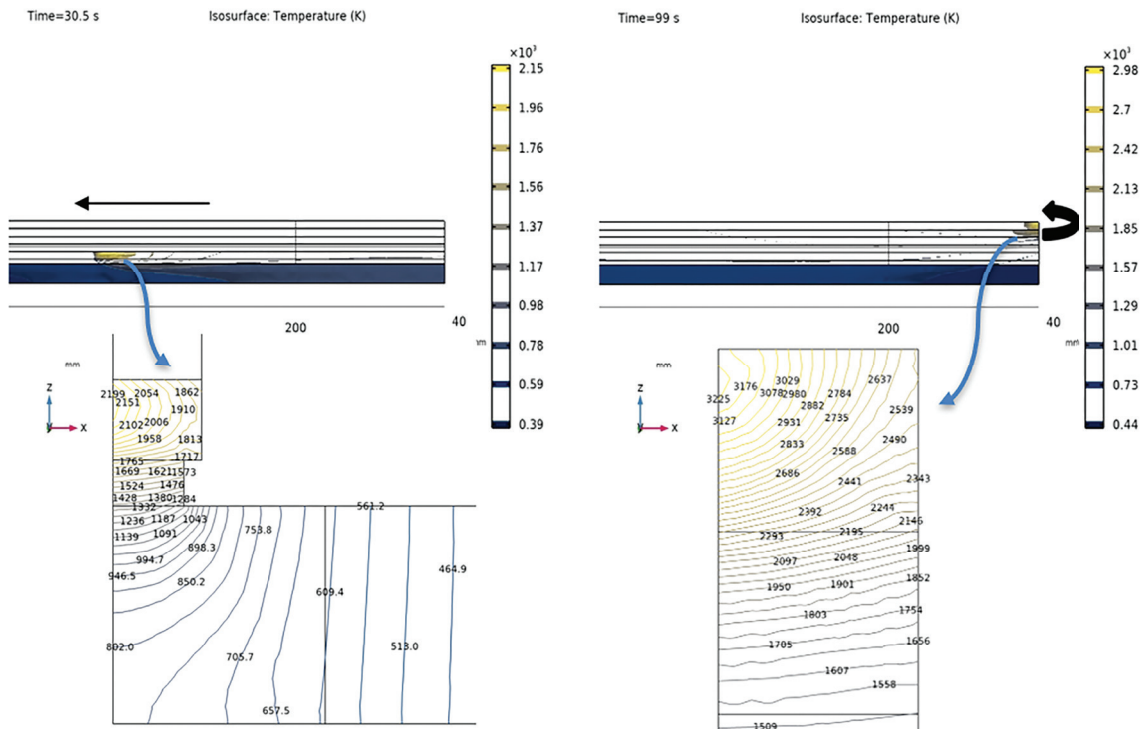


Fig. 10 — Side view of the isothermal contours at 30.5 s and 99 s with 2D cross-sectional labeled values of isotherms.

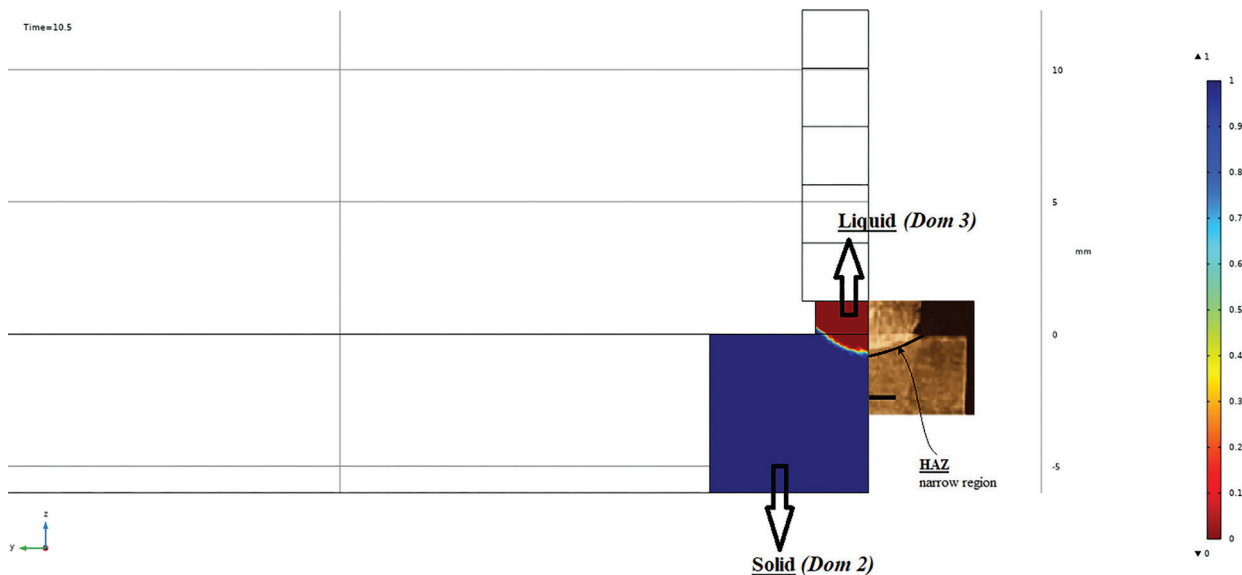


Fig. 11 — The left half section of the phase change simulation and the corresponding right half section of the micrograph, substrate, and first layer.

sidered in the interest of time, as the Z_0 value for the upper layers was assumed to be the same as the second layer (Table 11). This profile was extracted from a moving point in an FEM simulation in COMSOL, corresponding to the motion of the pyrometer in the experiment. The black dashed line shows the experimental temperature measured by the pyrometer mounted on the torch (see Fig. 5). Note that the initial few sec-

onds of the LumaSense curve (by InfraWin software), during and before which a preheating phase up to 90°C occurred, were excluded from the plot (visible on the right side of the figure) to enable an accurate comparison with the simulation. Both curves, therefore, start immediately after the torch is turned on. A real-time 3D temperature contour corresponding to the deposition at $t = 479.5$ s is also depicted on the curve.

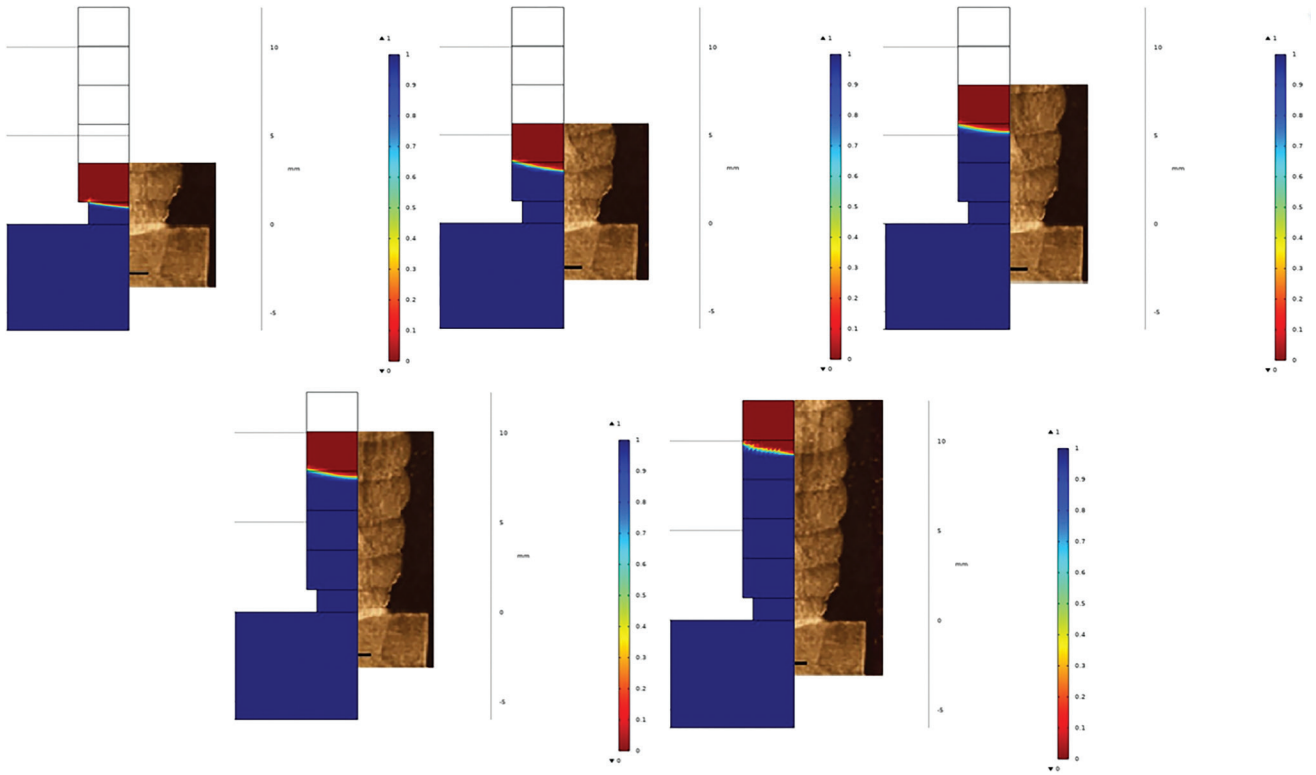


Fig. 12 — The left half section of the phase change simulation and the corresponding right half section of the micrograph, upper layers (2nd through 6th).

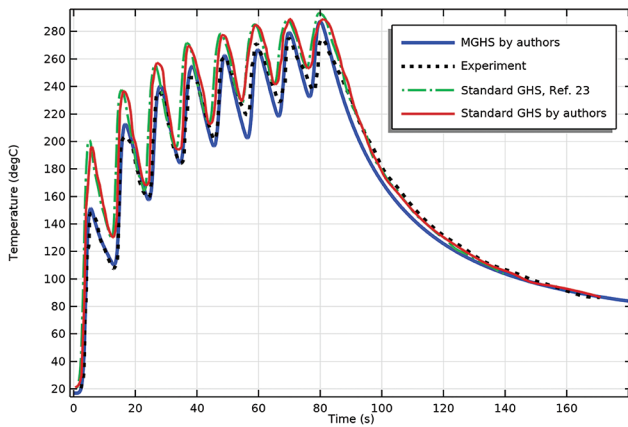


Fig. 13 — Experimental and simulated temperature profiles for WAAM - AA5356 Aluminum by Chergui et al. (Ref. 23) compared to the proposed MGHS and standard GHS by the authors.

The heat transfer coefficient in boundary conditions for the region within and around the weld pool was set on $h_{weld} = \varepsilon\sqrt{T}/5$ by parametric study and similarly far from the weld $h_{nonweld} = 2\varepsilon T/7$. The mean relative error is 3%.

Discussion

Tuning the Process and Heat Source Parameters

In this study, the primary validation of the MGHS was started with DSS, and a combination of parametric study and optimization was applied to achieve the MGHS parameters. For other alloys, parameters were fit using a parametric study without optimization. The following tuning process of parameters presents the procedure only for the DSS model. After multiple simulation runs, it was found that the combined parameter nI for the first layer, with a value of 165.85, resulted in the minimum mean relative error (MRE) of 13.72% between the experimental and numerical temperature curves. The upper layers were assigned to the same amperage as the experimental current (155 A), and the efficiency was set to 0.62. The error of 13.72% also corresponds to the standard Goldak model (Fig. 6).

Next, to reduce the error, a detailed parametric study was conducted to adjust A, B in $h_{weld} = A\varepsilon T^B$ and C, D in $h_{nonweld} = C T^D$ using the mesh described above; Ultimately, Equations 5 and 6 were obtained, and the error dropped to 2.75%.

The final equation of the combined convective heat transfer coefficient for the weld surfaces:

$$h_{weld} = 10^{-4} \varepsilon T^2 \quad (5)$$

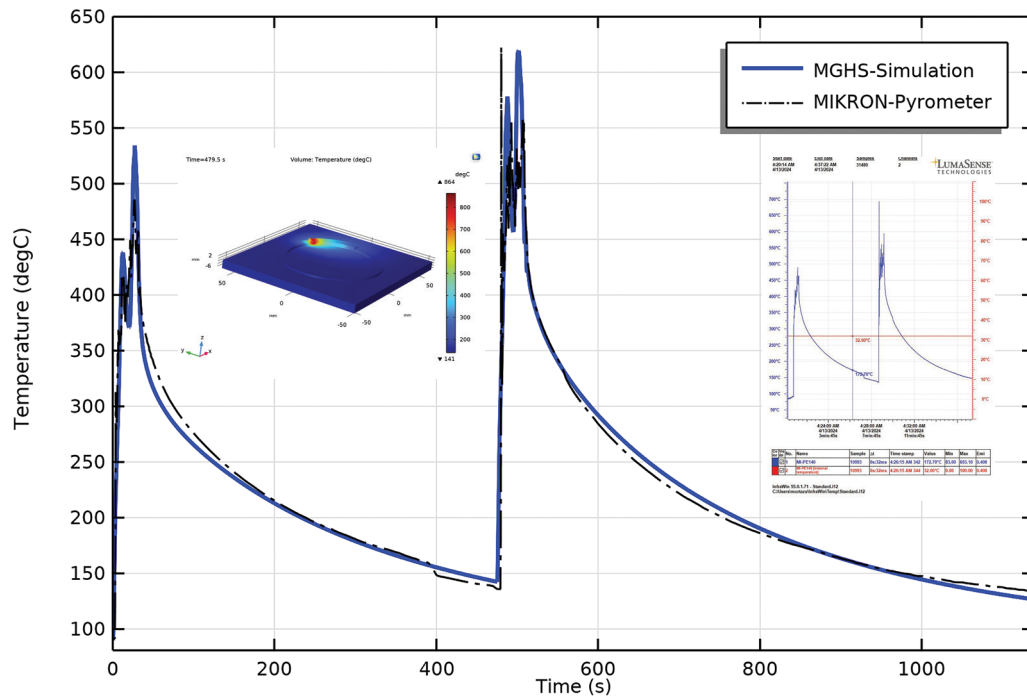


Fig. 14 — Temperature profiles from simulation (solid blue line) and experiment (black dashed line), starting after arc initiation.

and the convective coefficient for the non-welded surfaces becomes:

$$h_{non-welded} = 0.5 \sqrt{T} \quad (6)$$

Both h_{weld} and $h_{nonwelded}$ were scaled by T_{ref} (i.e., T/T_{amb}) in all parametric studies to avoid unit disagreement; T_{amb} cancels out when raised to a power and is absorbed into the constants.

User-defined functions (UDFs) were coded for both convective heat transfer and radiation to ensure that the moving boundary conditions are well-defined, meaning that once the wire is deposited and solidified, the surface underneath neither emits thermal radiation nor experiences convection. In contrast, the free surfaces being deposited involve both convection and radiation.

Using the constraints in Table 3, along with the boundary conditions and adjusted parameters, the correction term Z_o and the heat source parameter c_r were tuned using the parameter estimation method. A threshold of 9% error was set using mesh type 1 (Fig. 3). Table 11 includes their final values. At this point, the error dropped to 8.96%.

To achieve a lower error, some new parametric studies were implemented, and the final heat source parameters are given in Table 12. A couple of errors, as shown in Fig. 6 (7.34% and 5.98%), occur during this step.

A small number of simulations were implemented using a fine mesh as mesh type 2 for the final confirmation of parameters, which is much finer than the previous one, consisting

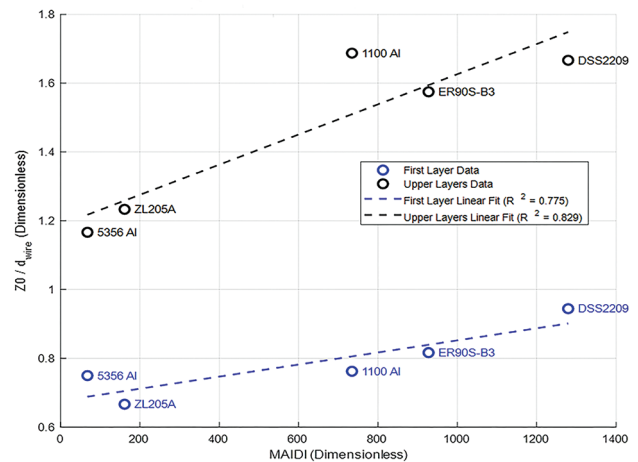


Fig. 15 — Curve fit of MAIDI vs. (Z_o/d_{wire}) for the first and upper layers of five alloys.

of 469,863 domain elements, 62,250 boundary elements, and 5400 edge elements. It includes a tetrahedral mesh with a maximum size of 0.5 mm (based on mesh study; for a few smaller values, no considerable reduction of error was observed, while increasing the computational cost) for the layers and the substrate region around the HAZ, where the mesh is also biased around the edges. The number of degrees of freedom solved for the final simulation is 519,002 plus 3,940,631 internal degrees of freedom (DOFs). The computational time for the last simulation, including the thermal

Table 13 – Main Parameters of the Dimensionless Number MAIDI

Parameter	Description
TS	Travel speed (m/s)
WFS	Wire feed speed (m/s)
k	Thermal conductivity of the material (W/(m·K))
α	Thermal diffusivity of the material (m ² /s)
η	Arc efficiency
V	Voltage (V)
I	Current (A)
ΔT	$T_{\text{melt}} - T_{\text{amb}}$ (K)
d_{wire}	Wire diameter (m)

expansions on a computer configured with an Intel® Core™ i9-7980XE CPU @ 2.60GHz, 36 processors, and 64GB RAM, was 32 h and 10 min. The majority of the simulations using mesh type 1 did not exceed a couple of hours. Figure 7 illustrates more consistency with the trend that the experiment follows. It shows the validated temperature profiles with MRE of 2.75% for the MGHS, which was compared to the temperature recorded by the thermocouple at the location (45 mm, 0, –6 mm) at the bottom, the same location measured by Hejripour et al. (Ref. 6).

The root mean squared error (RMSE) of the temperature (Eq. 7) in the MGHS was achieved 26.6 K, which is quite satisfactory. All the intermediate errors were calculated through a MATLAB MRE code between the thermocouple's temperature data and the corresponding numerical temperature, which adjusted the SNOPT optimality tolerance in COMSOL.

$$\text{Temperature Mean Relative Error (MRE)} = \frac{1}{n} \sum_{i=1}^n \frac{|T_{1,i} - T_{2,i}|}{T_{2,i}} \quad (7)$$

n is the total number of data points; $T_{1,i}$ is the interpolated value from the thermocouple's measurements; and $T_{2,i}$ is the corresponding interpolated simulated temperature

Temperature Contours and Isothermal Plots for DSS

Figure 8 captures the dynamic heat source movement and element activation as the wire is fed into the weld pool. It was

observed that the temperature distribution around the weld pool remains stable during the deposition process, except near the two end boundaries. The shapes of this distribution in the first and second layers, representing the entire process, are shown in Figs. 8A and B. A similar distribution in the middle of the sixth layer with some highlight of the deposition in the weld pool (which visualizes the element activation) is presented in Fig. 8C. The temperature distribution at the end of the sixth layer, where the arc turns around and begins to move in the opposite direction on the upper layer, is depicted and highlighted with the actual part of the samples in Fig. 8D, demonstrating a higher re-melting rate of the lower layer, which causes a slight downward flow of the weld. This is also observed in Fig. 10 at $t = 99$ s.

The isothermal surfaces (Figs. 9 and 10) demonstrate how powerful the element activation technique is for visualization of the moving heat source in WAAM while depositing the material on the previous layer, without solving the expensive Navier-Stokes equations of local fluid flow of the WP. Additionally, the MGHS model successfully visualized the weld pool dynamics without requiring fluid flow analysis, as shown in Figs. 9 and 10, appearing highly realistic. They also show the localized heat concentration near the deposition and its dissipation farther away is evident at different times. Cooler regions expand gradually in later frames, indicating progressive solidification. For the isothermal contours in Fig. 9, corresponding 2D isothermal cross sections with more contour levels close to the moving weld pool are also given below each one. Also, in Fig. 10, more detailed 2D cross-sectional isotherms associated with the weld pool are magnified at each time with labeled values of the isotherms.

Phase Change Validation Based on the Cross-Sectional Macrograph for DSS

A significant correlation was observed (Figs. 11 and 12), once the maximum liquidus area and mushy zone (Refs. 22, 34) were reached at the location where the heat source just passed by, between the real-time phase change and the cross-sectional macrograph of the fully solidified sample. This type of interlayer boundary shape analysis without the local fluid flow simulation (Ref. 35) was not discussed in the literature to validate the thermal mode. Solid-liquid phase change was implemented in COMSOL Multiphysics using the Heat Transfer Module, with effective specific heat and effective thermal conductivity defined as functions of phase fractions to model the solid-liquid transition and latent heat effects.

The cross-sectional 2D plot of phase change revealed a concave curve at the interlayer boundary formation in the weld pool (WP) as the heat source progressed (Fig. 11). This was compared to the cross-sectional macrograph of the fully solidified weld for interlayer boundary validation. The achieved consistency demonstrates that the real-time solid-liquid phase transition and subsequent solidification patterns are intricately linked. The figure also illustrates the narrow mushy zone in the 2205 DSS substrate in GMAW. It should be noted that near the front and back insulated surfaces (at $x = 0$ and $x = 250$ mm), there is a region in which more remelting of lower layers happens, so this phase change validation is more accurate closer to the middle of the WoW.

Table 14 — Z_o and MAIDI Number for Five Different Alloys

Parameter	DSS2209	5356 Al	1100 Al	ER90S-B3	ZL205A
TS (mm/s)	12	10	7.54	10	8
WFS (mm/s)	147	83.33333	190.5	133.3333	100
k_{avg} (W/mm·K)	18e-3	130e-3	215e-3	33e-3	140e-3
η	0.62	0.83	0.77	0.85	0.88
V_{avg} (V)	25	12	25	19	13.2
I_{avg} (A)	155	80	160	255	107
ΔT [K]	1420	628	616	1495	573
d_{wire} (mm)	0.9	1.2	0.8	1.2	1.2
Z_o , 1st layer (mm)	0.85	0.9	0.61	0.98	0.8
Z_o , upper layer (mm)	1.5	1.4	1.35	1.89	1.48
Experiment	Ref. 6	Ref. 23	Authors	Ref. 36	Ref. 37
MAIDI	1.2937e+03	6.7777e+01	7.2435e+02	7.6519e+02	1.6139e+02

Validation of the Proposed MGHS on AA5356 Aluminum WoW

The MGHS model developed in this paper, incorporating the novel arc origin offset parameter Z_o , was also validated through numerical simulations of WAAM-AA5356, demonstrating significant improvements over the standard Goldak model. The WAAM process parameters and the Goldak parameters for the AA5356 Aluminum are given in Tables 8 and 9. The temperature-dependent material properties were also imported in the same manner as those used by Chergui et al. (Ref. 23) in their simulations.

The standard Goldak model, combined with the new finite element method (FEM) proposed by Chergui et al., reported an approximate average error of 10% between experimental and simulated temperature curves. However, the minimum error obtained in the standard Goldak heat source conducted by the authors of this paper is 8%, which must be due to the temperature-dependent convective BCs. By integrating the Z_o parameter (of the MGHS model), the proposed model reduced the MRE to 3.4%, highlighting superior accuracy in capturing thermal phenomena as seen in Fig. 13.

Validation of the MGHS on AA1100 ToW Using a Moving Pyrometer

The agreement between the simulated and experimental temperature curves with 3% MRE is highly acceptable. The moving pyrometer monitoring approach proves to be more reliable, as it continuously monitors the temperature at almost countless locations along the path, unlike fixed thermocouples, which are limited to a single point or just a few. This improves the versatility and diversity of the data in the sensitivity analysis, which results in a user-friendly curve fit to pick the Z_o value.

Incorporating a Novel Dimensionless Number into Sensitivity Analysis

This section presents a targeted, preliminary sensitivity evaluation of the MGHS model's accuracy with respect to the Z_o parameter. Instead of performing a classical one-at-a-time sensitivity analysis, where process parameters and material properties are varied individually, the authors introduced a novel dimensionless number derived from the nondimensionalized heat transfer equation (Eq. 1) that combines these factors in a physically meaningful way. Classical sensitivity analysis can be time-consuming and may overlook interactions between parameters, especially since WAAM process parameters are mostly interdependent.

Varying material properties one at a time isn't practical, as it essentially involves changing the alloy, given that simulations are implemented using temperature-dependent material properties. Additionally, validating one-at-a-time sensitivity simulations through extensive experiments for each combination would be very costly. In contrast, the proposed dimensionless number captures these complex interactions more effectively. Using this number, the Z_o values from five different alloys and their respective process parameters were mapped into a single empirical curve fit, providing a practical and efficient tool for selecting Z_o .

MAIDI Number

Nondimensionalizing the heat transfer equation (Eq. 1) for WAAM yields a dimensionless group capturing process-dependent thermal behavior. The characteristic scales are: $T^* = (T - T_{amb})/t_{scale}$, $t_{scale} = \Delta T = T_{melt} - T_{amb}$ as the temperature scale, $t^* = t/t_{scale}$, $t_{scale} = L^2/\alpha$ which is the time scale, and the length scale is chosen a process-dependent characteristic scale (This choice makes the dimensionless equation more representative of real welding [WAAM] conditions and is advantageous for scaling across different welding speeds and wire feed rates. It reflects the actual spatial scale of heat deposition in the weld pool, naturally incorporating the effects of travel speed and wire feed rate that primarily control the melt pool size and shape in additive layer geometry, and ensures the dimensionless equation accurately captures process-dependent thermal behavior. Obviously, a , b , c_{rf} are process dependent as well), $L_{scale} = TS (d_{wire}/WFS)$ representing the distance the torch travels while feeding one wire diameter; $d_{wire}/WFS =$ time to feed one wire diameter. Then, $x^* = x/L_{scale}$, $y^* = y/L_{scale}$, $z^* = z/L_{scale}$, $a^* = a/L_{scale}$, $b^* = b/L_{scale}$, $Z_o^* = Z_o/L_{scale}$, $c_{rf}^* = c_{rf}/L_{scale}$, $Q_{arc} = \eta VI$, $x' = x - vt$; v which is the travel speed and is taken as the characteristic velocity (Ref. 38) in Peclet number: $Pe = vL/\alpha$. Equations 2 and 3 are the heat source equations, which are combined (to avoid rewriting the right-hand side) considering $c = c_r$ or $c = c_f$ and $f = f_r$ or $f = f_f$ so only one term on the right-hand side of Equation 1 appears. So, let's start with the right-hand side,

$$q_{arc} = \frac{6\sqrt{3} f \eta VI}{abc\sqrt{\pi^3}} \exp \left(-3 \left(\frac{(x - v(t))^2}{c^2} + \frac{y^2}{a^2} + \frac{(z - Z_o)^2}{a^2} \right) \right)$$

$$= \frac{6\sqrt{3} f \eta VI}{L^3 a^* b^* c^* \sqrt{\pi^3}}$$

$$\exp \left(-3 \left(\frac{(Lx^* - v(\frac{L^2}{\alpha}) t^*)^2}{L^2 c^{*2}} + \frac{L^2 y^{*2}}{L^2 a^{*2}} + \frac{L^2 (z^* - Z_o^*)^2}{L^2 b^{*2}} \right) \right)$$

$$= \frac{6\sqrt{3} f \eta VI}{L^3 a^* b^* c^* \sqrt{\pi^3}} \exp$$

$$\left(-3 \left(\frac{L^2 (x^* - v(\frac{L}{\alpha}) t^*)^2}{L^2 c^{*2}} + \frac{L^2 y^{*2}}{L^2 a^{*2}} + \frac{L^2 (z^* - Z_o^*)^2}{L^2 b^{*2}} \right) \right)$$

$$= \frac{6\sqrt{3} f \eta VI}{L^3 a^* b^* c^* \sqrt{\pi^3}}$$

$$\exp \left(-3 \left(\frac{(x^* - (Pe)t^*)^2}{c^{*2}} + \frac{y^{*2}}{a^{*2}} + \frac{(z^* - Z_o^*)^2}{b^{*2}} \right) \right)$$

So, the whole term is nondimensional except VI/L^3 , with units of $[W/m^3]$. Now, let's apply the same approach to the left-hand side of Equation 1. Knowing thermal diffusivity is $\alpha = k/(\rho C_p)$, so

$$\rho C_p = \frac{k}{\alpha}$$

$$\rho C_p \left(\frac{\partial(T^* \Delta T + T_{amb})}{\partial(t^* \frac{L^2}{\alpha})} \right) - \frac{k \nabla^{*2}(T^* \Delta T + T_{amb})}{L^2}$$

$$= \frac{k}{\alpha L^2} \left(\frac{\partial T^*}{\partial t^*} \right) \Delta T - \frac{k \nabla^{*2} T^*}{L^2} \Delta T = \frac{k \Delta T}{L^2} \left(\frac{\partial T^*}{\partial t^*} - \nabla^{*2} T^* \right)$$

The term $k \Delta T / L^2$ has the same dimension as VI/L^3 .

Now Equation 1 becomes:

$$\frac{k \Delta T}{L^2} \left(\frac{\partial T^*}{\partial t^*} - \nabla^{*2} T^* \right)$$

$$= \frac{6\sqrt{3} f \eta VI}{L^3 a^* b^* c^* \sqrt{\pi^3}} \exp \left(-3 \left(\frac{(x^* - (Pe)t^*)^2}{c^{*2}} + \frac{y^{*2}}{a^{*2}} + \frac{(z^* - Z_o^*)^2}{b^{*2}} \right) \right)$$

which can be written as:

$$\frac{\partial T^*}{\partial t^*} - \nabla^{*2} T^* = \frac{L^2}{k\Delta T} \frac{6\sqrt{3} f \eta V I}{L^3 a^* b^* c^* \sqrt{\pi^3}} \exp \left(-3 \left(\frac{(x^* - (Pe)t^*)^2}{c^{*2}} + \frac{y^{*2}}{a^{*2}} + \frac{(z^* - Z_0^*)^2}{b^{*2}} \right) \right)$$

Let's plug in let's plug in $L = L_{scale}$

$$\begin{aligned} \frac{\partial T^*}{\partial t^*} - \nabla^{*2} T^* &= \frac{1}{k\Delta T} \frac{6\sqrt{3} f \eta V I}{(TS(d_{wire}/WFS)) a^* b^* c^* \sqrt{\pi^3}} \\ &\exp \left(-3 \left(\frac{(x^* - (Pe)t^*)^2}{c^{*2}} + \frac{y^{*2}}{a^{*2}} + \frac{(z^* - Z_0^*)^2}{b^{*2}} \right) \right) = \end{aligned}$$

$$\begin{aligned} \frac{1}{k\Delta T} \left(\frac{\eta V I}{TS} \right) \left(\frac{WFS}{d_{wire}} \right) \frac{6\sqrt{3} f}{a^* b^* c^* \sqrt{\pi^3}} \\ \exp \left(-3 \left(\frac{(x^* - (Pe)t^*)^2}{c^{*2}} + \frac{y^{*2}}{a^{*2}} + \frac{(z^* - Z_0^*)^2}{b^{*2}} \right) \right) \\ \rightarrow \frac{\partial T^*}{\partial t^*} - \nabla^{*2} T^* = \left[\left(\frac{WFS}{d_{wire}} \right) \left(\frac{\eta \frac{V \cdot I}{TS}}{k} \right) / (\Delta T) \right] q^* \end{aligned}$$

Which is the nondimensional heat transfer equation, so $q^* = q_{arc}/q_{scale}$, $q_{scale} = (WFS/d_{wire})(\eta \cdot V \cdot I / TS / K) / (\Delta T)$, which is rewritten as

$$\begin{aligned} \frac{\partial T^*}{\partial t^*} - \nabla^{*2} T^* \\ = (MAIDI) q^* (x^*, y^*, z^*, Z_0^*, t^*, a^*, b^*, c_r^*, c_f^*, Pe) \end{aligned} \quad (8)$$

This new dimensionless number which can be extended to a variety of welding processes in both experiments and simulation, is termed MAIDI (Mortazavi-Aidun Interaction Deposition Index).

$$MAIDI = \left(\frac{WFS}{d_{wire}} \right) \left(\frac{\eta \frac{V \cdot I}{TS}}{k} \right) / (\Delta T) \quad (9)$$

The index in Equation 9 varies directly with process parameters that govern heat input and deposition rate. For a given filler wire alloy (with fixed thermal properties), increasing the wire feed rate (WFS) raises the heat input per unit length,

enlarging the melt pool and increasing the index. In contrast, increasing travel speed (TS) decreases the heat input per unit length, resulting in smaller melt pools and lower index values. For a fixed alloy or wires with very similar thermal properties, reducing layer height increases MAIDI if driven by higher TS (enhancing heat accumulation via faster deposition) but decreases MAIDI if due to lower WFS (reducing heat input). Thus, for a fixed alloy, the index effectively captures how process conditions influence the thermal field and melt pool geometry in WAAM. When different filler wire alloys are employed, variations in thermal conductivity, melting point, and thermal diffusivity modify the effective heat distribution and solidification behavior. Consequently, the same process parameters may yield different index values, and different process parameters and filler alloys may require further discussion, as their combined effects influence the index behavior. Some implications of these variations can be roughly inferred from Fig. 15. Note that extremely low or high WFS or TS values (outside the typical operational range for WAAM) may cause arc instability or defects (e.g., porosity, cracks), limiting the practical range of MAIDI. The parameters are defined in Table 13.

The terms have specific physical meanings:

1. (WFS/d_{wire}) : the ratio of wire feed speed to wire diameter. This term quantifies the rate at which material is delivered per size of wire.

2. $(\eta(V \cdot I / TS)) / k$ represents the ratio of the effective heat input to the diffusion capability of the material.

Division by ΔT serves as thermal normalization, indicating the minimum temperature difference to melt. The MAIDI number was used to calibrate the Z_0 parameter proposed in the MGHS model for different material properties and process parameters. To have diverse representatives of alloys that are capable of being fabricated by WAAM, a couple of MGHS models on two experiments published by FWC Farias et al. (Ref. 36) and Z. Wang et al. (Ref. 37) on ER90S-B3 and ZL205A were also simulated with errors of 3.2% and 3.9%, respectively. Table 14 presents the values for the parameters from Table 13 for five alloys simulated by the MGHS model and validated by experiments conducted by the authors or published in the literature.

The MATLAB numerical analysis of the MAIDI dimensionless number against the normalized stand-off distance (Z_0 / d_{wire}) for these five alloys, (DSS2209, 5356 Al, 1100 Al, ER90S-B3, and ZL205A) demonstrates a robust correlation through linear curve fitting as seen in Fig. 15. Linear fits, provide R^2 values of 0.78 and 0.83 for the first and upper layers, respectively, however, incorporating additional experiments and corresponding MGHS models, of course, could further refine the curve fit, enhancing the accuracy of Z_0 predictions. Both the first layer (blue) and the upper layers (black) show a positive correlation between the dimensionless MAIDI number and Z_0 / d_{wire} . All the users need is to calculate the MAIDI dimensionless number (Eq. 9), representing the metal and process parameters, and pick the value for Z_0 / d_{wire} from Fig. 15, and obtain the Z_0 value.

It is also observed that the steel alloys exhibited higher MAIDI numbers, accompanied by proportionally higher Z_0 / d_{wire} ratios compared with the aluminum alloys, with small statistical deviations for 5356 Al in the first layer and 1100 Al in the upper layers, as discussed previously.

Conclusion

This study developed an MGHS model for GMAW-based WAAM, introducing an arc origin offset parameter (Z_o) to enhance the precision of double ellipsoidal heat source modeling. Unlike the standard Goldak heat source equations, which are basically for a single weld bead, the proposed model incorporates Z_o . This proved essential for more accurate and robust tuning of heat source parameters across the WAAM layers, while the standard GHS in WAAM cases has yielded unrealistic parameter ranges, artificially high temperatures, or inconsistent interlayer boundaries and larger errors. By employing DSS and AA5356 aluminum experimental temperature data, the MGHS significantly lowered errors compared to the standard Goldak model. The MGHS's error for the DSS model dropped to 2.75% compared to a surface Gaussian heat source reported error of 12% (Ref. 6), and for AA5356 to 3.4% compared to 10% standard GHS's error. For DSS, the model's phase change predictions aligned closely with cross-sectional macrographs, proving the successful interlayer boundary shape analysis. Moreover, the MGHS model, validated through an AA100 aluminum tube experiment conducted by the authors using a moving pyrometer, achieved a 3% error in temperature profile predictions. Also, a couple of MGHS models validated by two experiments conducted by F.W.C. Farias et al. (Ref. 36) and Z. Wang et al. (Ref. 37) on ER90s-B3 and ZL205A proved accurate with errors of 3.2% and 3.9%, respectively. It was overall observed that a value between $0.4\times$ (the height of the first layer) and $0.6\times$ (the height of the first layer) for the first layer, and a value greater than $0.7\times$ (the height of the upper layers) for the upper layers, yields an error of less than 10%. As for the shape of the MGHS heat source, apart from the centroid of the double-ellipsoidal heat source that is shifted upward from the substrate surface (either the Base Metal or previous layer), C_r and C_f . The semi-ellipsoidal parameters along travel direction in the MGHS model showed greater values than the corresponding values of the standard Goldak model (schematically depicted in Fig. 4D), and a (along the y-axis, perpendicular to the travel direction) was obtained slightly less in MGHS than that of the standard Goldak model for all five MGHS validations. However, b , along the build direction should be tuned and could be the same, slightly higher, or lower.

The novel MAIDI dimensionless number effectively enables a curve fitting for the Z_o parameter across five distinct alloys that results in R^2 values of 0.78 and 0.83, respectively. This curve provides researchers with appropriate values for Z_o . Including more experiments and MGHS models could improve the curve fitting process and increase the accuracy of Z_o predictions across various alloys with different process parameters.

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Appendix A

Summation of Integration of Heat Source Equations

The summation of integration of the heat source equations (Eqs. 2 and 3) must equal the net power input to hold the energy conservation. The main difference between the MGHS equations and the standard GHS equations is the Z_o parameter, which takes two constant values in this paper, as explained in Fig. 4: one for the first layer and the other for the upper layers. Of course, even with different values of Z_o for each layer, this parameter remains constant across the layer. So, $q_r(x, y, z, t)$ and $q_f(x, y, z, t)$ are already defined in Equations 2 and 3. Also take $x' = x + v(\tau - t)$, and $z' = z - z_o$, then $dx' = dx$, $dz' = dz$. Also, from the standard Goldak model (Ref. 1), $f_r + f_f = 2$. Let's take constant $C_q = (6\sqrt{3}Q_{arc})/(abc\sqrt{\pi^3})$ then

$$\begin{aligned} J &= \iiint (q_r(x, y, z, t) + q_f(x, y, z, t)) dx dy dz \\ &= C_q \left[\int_0^{+\infty} e^{-3\left(\frac{z'^2}{b^2}\right)} dz' \int_{-\infty}^{+\infty} e^{-3\left(\frac{y^2}{a^2}\right)} dy \right. \\ &\quad \left. \left(f_r \int_{-\infty}^0 e^{-3\left(\frac{x'^2}{c^2}\right)} dx' + f_f \int_0^{+\infty} e^{-3\left(\frac{x'^2}{c^2}\right)} dx' \right) \right] \end{aligned}$$

knowing

$$\int_{-\infty}^0 e^{-(x'^2)} dx' = \frac{\sqrt{\pi}}{2},$$

$$\int_0^{+\infty} e^{-(x'^2)} dx' = \frac{\sqrt{\pi}}{2},$$

$$\int_{-\infty}^{+\infty} e^{-(y'^2)} dy = \sqrt{\pi}$$

and

$$\int_0^{+\infty} e^{-(z'^2)} dz' = \frac{\sqrt{\pi}}{2}$$

the integrations result in

$$\int_{-\infty}^0 e^{-3(\frac{y'^2}{c^2})} dx' = \frac{c}{2\sqrt{3}}\sqrt{\pi}, \int_0^{+\infty} e^{-3(\frac{y'^2}{c^2})} dx' = \frac{c}{2\sqrt{3}}\sqrt{\pi},$$

$$\int_{-\infty}^{+\infty} e^{-3(\frac{y'^2}{a^2})} dy = \frac{a}{\sqrt{3}}\sqrt{\pi},$$

and

$$\int_0^{+\infty} e^{-3(\frac{z'^2}{b^2})} dz' = \frac{b}{2\sqrt{3}}\sqrt{\pi}.$$

$$\begin{aligned} J &= C_q \left(\frac{c}{2\sqrt{3}}\sqrt{\pi} \right) \left(\frac{a}{\sqrt{3}}\sqrt{\pi} \right) \\ &\quad \left(f_r \frac{b}{2\sqrt{3}}\sqrt{\pi} + f_f \frac{b}{2\sqrt{3}}\sqrt{\pi} \right) \\ &= C_q \left(\frac{abc}{12\sqrt{3}}\sqrt{\pi^3} \right) (f_r + f_f); \end{aligned}$$

now, by plugging in

$$C_q = \frac{6\sqrt{3} Q_{arc}}{abc\sqrt{\pi^3}}$$

$$\begin{aligned} J &= \frac{6\sqrt{3} Q_{arc}}{abc\sqrt{\pi^3}} \left(\frac{abc}{12\sqrt{3}}\sqrt{\pi^3} \right) (f_r + f_f) \\ &= Q_{arc} \end{aligned}$$

therefore, $J = Q_{arc}$ which is the net power input.

Appendix B

Heat Source Parameters for Five Different Alloys

The MGHS parameters for each filler wire are given in the tables below. Their variations with travel speed and wire feed speed are also depicted in each case.

As WFS increases (as shown in the tables using arrows), MGHS parameters increase to reflect larger melt pools. As TS increases, they decrease due to reduced energy input per length. Therefore, adjusting these two parameters together matters the most when the goal is to keep the same weld bead (additive layer's) geometry for the same filler wire or a wire with similar composition and thermal properties to achieve the same heat source parameters (a, b, c_r, c_f, Z_0) in thermal

Table B.1 — DSS ToW's MGHS Parameters

	Q (W)	η	a	b	C _f	C _r	Z ₀	f _r	f _f
1st Layer	3875	0.62	1.7 (mm) = (0.067 in.)	1.2 mm (0.047 in.)	1.9 mm (0.075 in.)	9 mm (0.354 in.)	0.85 mm (0.033 in.)	1.25	0.75
Upper Layers	same	same	2 (mm) (0.079 in.)	1.5 mm (0.059 in.)	2.9 mm (0.114 in.)	12 mm (0.472 in.)	1.5mm (0.0591 in.)	same	same
If TS ↑			↓	↓	↓	↓	↓		
If WFS ↑			↑	↑	↑	↑	↑		
Local Convective Heat Transfer Coefficient, h _{weld}				h _{weld} = 10 ⁻⁴ ε T ²			[W/m ² ·K]		
Convective Heat Transfer Coefficients, h _{nonwelded}				h _{nonwelded} = 0.5 sqrt(T)			[W/m ² ·K]		
Gas Flow Rate*					45 ft ³ /h				

As already mentioned, both h_{weld} and $h_{nonwelded}$ were scaled by T_{ref} (i.e., T/T_{amb}) in all parametric studies to avoid unit disagreement; T_{amb} cancels out when raised to a power and is absorbed into the constants.

* Gas flow rate impacts h_{weld} ; the higher the flow rate, the higher the convective heat transfer coefficient.

modelling. So, in this regard, apart from the qualitative trend of how the fitting parameters change across the process parameters, most importantly, WFS and TS were varied for the 1100 Al experiment to keep the bead geometry and accordingly the heat source parameters (a , b , c_r , c_f , Z_0) unchanged as much as possible. The result is that increasing WFS by 100 in./min (from 450 to 550 in./min) required an increment of approximately 1.1 in./min in TS as well to maintain the same

bead geometry for each layer, and accordingly the fitting parameters. The MGHS model was also validated with a low error of approximately 3.5% corresponding to this variation.

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Table B.2 — AA5356 WoW's MGHS Parameters

	Q (W)	η	a	b	C_f	C_r	Z_0	f_r	f_f
1st Layer	960	0.83	3 (mm) (0.118 in.)	3 (mm) (0.118 in.)	3 (mm) (0.118 in.)	6.5 (mm) (0.256 in.)	0.9 (mm) (0.035 in.)	1.25	0.75
Upper Layers	same	same	same	0.9 (mm) (0.035 in.)	4 (mm) (0.157 in.)	8 (mm) (0.315 in.)	1.4 (mm) (0.055 in.)	same	same
If TS ↑			↓	↓	↓	↓	↓		
If WFS ↑			↑	↑	↑	↑	↑		
Local Convective Heat Transfer Coefficient, h_{weld}							$h_{\text{weld}} = \varepsilon\sqrt{T}/15$		
Convective Heat Transfer Coefficients, $h_{\text{nonwelded}}$							$h_{\text{nonwelded}} = 2\varepsilon T/7$		
Gas Flow Rate							27.5 ft ³ /h		

Table B.3 — AA1100 ToW's MGHS Parameters

	Q (W)	η	a	b	C_f	C_r	Z_0	f_r	f_f
1st Layer	3080	0.77	4.2 (mm) (0.165 in.)	0.55 (mm) (0.022 in.)	5.5 (mm) (0.217 in.)	12.5 (mm) (0.492 in.)	0.61 (mm) (0.024 in.)	1.2	0.8
Upper Layers	same	same	same	0.35 (mm) (0.014 in.)	6.5 (mm) (0.256 in.)	12.7 (mm) (0.5 in.)	1.35 (mm) (0.053 in.)	same	same
If TS ↑			↓	↓	↓	↓	↓		
If WFS ↑			↑	↑	↑	↑	↑		
Local Convective Heat Transfer Coefficient, h_{weld}						$h_{\text{weld}} = \varepsilon\sqrt{T}/5$			
Convective Heat Transfer Coefficients, $h_{\text{nonwelded}}$						$h_{\text{nonwelded}} = 2\varepsilon T/7$			
Gas Flow Rate						35 ft ³ /h			

Table B.4 — ER90S-B3 WoW's MGHS Parameters

	Q (W)	η	a	b	C _f	C _r	Z ₀	f _r	f _f
1st Layer	4845	0.85	3 (mm) (0.126 in.)	0.9 (mm) (0.0256 in.)	3.4 (mm) (0.134 in.)	6.5 (mm) (0.256 in.)	0.98 (mm) (0.386 in.)	1.4	0.6
Upper Layers	same	same	3.2 (mm) (0.126 in.)	1.2 (mm) (0.0256 in.)	4.3 (mm) (0.169 in.)	8 (mm) (0.315 in.)	1.89 (mm) (0.074 in.)	same	same
If TS ↑			↓	↓	↓	↓	↓		
If WFS ↑			↑	↑	↑	↑	↑		
Local Convective Heat Transfer Coefficient, h_{weld}					$h_{weld} = 10^{-4} \varepsilon T^2$				
Convective Heat Transfer Coefficients, $h_{nonwelded}$					$h_{nonwelded} = 0.5 \sqrt{T}$				
Gas Flow Rate					42.4 ft ³ /h				

Table B.5 — ZL205A WoW's MGHS Parameters

	Q (W)	η	a	b	C _f	C _r	Z ₀	f _r	f _f
1st Layer	1412.4	0.88	4.2 (mm) (0.173 in.)	0.8 (mm) (0.0315 in.)	3.2 (mm) (0.126 in.)	6.3 (mm) (0.248 in.)	0.8 (mm) (0.0315 in.)	1.15	0.85
Upper Layers	same	same	4.4 (mm) (0.173 in.)	1.1 (mm) (0.0256 in.)	4.2 (mm) (0.165 in.)	8.1 (mm) (0.319 in.)	1.48 (mm) (0.058 in.)	same	same
If TS ↑			↓	↓	↓	↓	↓		
If WFS ↑			↑	↑	↑	↑	↑		
Local Convective Heat Transfer Coefficient, h_{weld}					$h_{weld} = \varepsilon \sqrt{T/4}$				
Convective Heat Transfer Coefficients, $h_{nonwelded}$					$h_{nonwelded} = 2\varepsilon T/5$				
Gas Flow Rate					38.1 ft ³ /h				